PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL AND R. P. BAKER.

[Send all Communications to B. F. FINKEL, Springfield, Mo.]

PROBLEMS FOR SOLUTION.

ALGEBRA.

455. Proposed by JOS. B. REYNOLDS, Lehigh University.

Solve for x_n (not in determinant form) the simultaneous equations,

$$\tfrac{4}{3}\,x_n + 2x_{n-1} + 2x_{n-2} \cdots 2x_4 + 2x_3 + 2x_2 + 2x_1 = g,$$

$$\tfrac{10}{3}\,x_n + \tfrac{22}{3}\,x_{n-1} + 8x_{n-2} \cdots 8x_4 + 8x_3 + 8x_2 + 8x_1 = 4g,$$

$$\tfrac{16}{3}\,x_n + \tfrac{40}{3}\,x_{n-1} + \tfrac{52}{3}\,x_{n-2} \cdots 18x_4 + 18x_3 + 18x_2 + 18x_1 = 9g,$$

$$\tfrac{22}{3}\,x_n + \tfrac{58}{3}\,x_{n-1} + \tfrac{82}{3}\,x_{n-2} + \tfrac{94}{3}\,x_{n-3} \cdots 32x_4 + 32x_3 + 32x_2 + 32x_1 = 16g,$$

$$\tfrac{28}{3}\,x_n + \tfrac{76}{3}\,x_{n-1} + \tfrac{112}{3}\,x_{n-2} + \tfrac{136}{3}\,x_{n-3} + \tfrac{148}{3}\,x_{n-4} \cdots = 25g,$$

. . . .

$$(\tfrac{1}{3} + 2n - 1)x_n + (\tfrac{1}{3} + 6n - 5)x_{n-1} + (\tfrac{1}{3} + 10n - 13)x_{n-2}$$
$$+ (\tfrac{1}{3} + 14n - 25)x_{n-3} \cdots (\tfrac{1}{3} + 2n^2 - 1)x_1 = n^2 g.$$

456. Proposed by PAUL CAPRON, U. S. Naval Academy.

If

$$S_{i,n} = \sum_{k=1}^{k=n-i+1} \frac{(i+k-1)!}{(k-1)!},$$

show that $S_{i,n}$ is equal to $1/(i+1)$ times the last term of $S_{i+1,n+1}$; as, for instance, that

$$S_{1,n} = 1 + 2 + \cdots + n = \frac{n}{2}(n+1),$$

that

$$S_{2,n} = 1\cdot 2 + 2\cdot 3 + \cdots + n(n+1) = \tfrac{1}{3}(n-1)n(n+1),$$

etc.

GEOMETRY.

486. Proposed by ARON INGVALE, Brooklyn, N. Y.

Does the following construction trisect an angle? With the vertex, O, of the given angle as center and with a radius R, describe a circle intersecting the sides of the given angle in A and B. With a radius $\tfrac{3}{4}R$, and center on OA, describe a circle tangent to the other circle at A and cutting the other side of the angle at E. At E draw a tangent to the last circle and produce it to meet the first circle at F. Draw FO. Then is angle BOF one-third of the angle BOA?

REMARK. Though the construction does not, of course, lead to the trisection of an angle in general, yet as a first approximation it is very good. This fact together with the fact that the construction is very simple, and that the proposer's demonstration that it does trisect the angle is very illusive, are the reasons for giving the problem a place in the MONTHLY.

EDITORS.

487. Proposed by H. B. PHILLIPS, Massachusetts Institute of Technology.

If segments from the vertices A and B of a triangle to the opposite sides are of equal length and divide the angles A and B (measured from AB) proportionally, the triangle is isosceles.

488. Proposed by ROGER A. JOHNSON, Western Reserve University.

If triangles are constructed on a given base, having the radii of the incircle and circumcircle in a constant ratio, determine the locus of the vertex. (Necessarily the constant ratio is not greater than $\frac{1}{2}$.)

CALCULUS.

405. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

Determine the greatest quadrilateral which can be formed with the four given sides a, b, c, and d taken in order.

406. Proposed by C. N. SCHMALL, New York City.

Given $f(x+h)+f(x-h)=f(x)\cdot f(h)$, determine by Taylor's theorem or otherwise the nature of the function f.

MECHANICS.

324. Proposed by H. S. UHLER, Yale University.

A rigid body of any shape is at rest in a neutral liquid which is also at rest and has an indefinitely great volume. The body is so situated that the free surface of the liquid is tangent to it at its highest point (or points). All the space above the liquid is filled with a neutral, stagnant fluid whose density is not greater than the density of the liquid. Show that the work done in raising (pure translation) the body very slowly until the interface of the two fluids is tangent to it at its lowest point (or points) is expressible by the formula $mgh - gV(\rho_1 h_1 + \rho_2 h_2)$, where $m \equiv$ mass of body, $V \equiv$ volume of body, $\rho_1 \equiv$ mean density of lower medium in the region involved, $\rho_2 \equiv$ density of upper medium, $h_1 \equiv$ distance of center of mass of displaced liquid below the free surface in the initial position of the body, $h_2 \equiv$ elevation of center of mass of displaced fluid above interface in final position of body, and $h = h_1 + h_2$. (Neglect surface-tension, adhesion, cavities in upper portion of body, etc. This problem arose in connection with a question concerning the raising of a dense object from the bottom of a harbor to the deck of a vessel.)

325. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

The lever of a testing-machine is c feet long, and is poised on a knife-edge a feet from one end and b feet from the other, and in a horizontal line, above which the beam is symmetrical. The beam is m inches deep at the knife-edge, and tapers uniformly to a depth of n inches at each end; the width of the beam is the same throughout its length. Find the distance of the center of gravity of the beam from the knife-edge.

NUMBER THEORY.

242. Proposed by NORMAN ANNING, Chilliwack, B. C.

Find a function of n which is equal to A_k when $n \equiv k \pmod{p}$, $k = 1, 2, 3, 4, \cdots, p$.

243. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

Determine the rational value of x that will render $x^3 + px^2 + qx + r$ a perfect cube. Apply the result to $x^3 - 8x^2 + 12x - 6$.

Below are given problems in Number Theory proposed between January, 1913, and January, 1915, for which no solutions have been received. Ten problems in this subject were proposed during 1915. For some of these, solutions have been received and others are doubtless under consideration by those interested. They are Nos. 227–236. While not neglecting these more recent ones, may we also have coöperation in clearing up the older list?